

Objective 22

Convert a repeating decimal number to a fraction

Example 1:

Convert $6.\overline{672}$ to a mixed number. Write the answer in simplest form.

STEP 1

Write an equation that shows that some number x is equal to the decimal $6.\overline{672}$.

$$x = 6.\overline{672}$$

STEP 2

Rewrite the equation so only the block of repeating digits is to the right of the decimal point. Multiply both sides of the equation from Step 1 by 10.

$$\begin{aligned} 10 \cdot x &= 10 \cdot 6.\overline{672} \\ 10x &= 66.\overline{72} \end{aligned}$$

STEP 3

Multiply both sides of the equation from Step 2 to create a new equation that has one block of the repeating digits to the left of the decimal point. The number $66.\overline{72}$ has two digits in the repeating block, so multiply each side of the equation by 10^2 , or 100.

$$\begin{aligned} 100 \cdot 10x &= 100 \cdot 66.\overline{72} \\ 1000x &= 6672.\overline{72} \end{aligned}$$

STEP 4

Subtract equivalent values from both sides of the equation found in Step 3 to eliminate the repeating digits. Use the fact that $10x = 66.\overline{72}$.

$$\begin{array}{r} 1000x = 6672.\overline{72} \\ - 10x = 66.\overline{72} \\ \hline 990x = 6606.00 \end{array}$$

STEP 5

Solve the new equation for x . Divide both sides of the equation by 990. Then simplify the improper fraction.

$$\begin{aligned} 990x &= 6606 \\ \frac{990x}{990} &= \frac{6606}{990} \\ x &= \frac{6606}{990} \\ x &= \frac{6606 \div 18}{990 \div 18} \\ x &= \frac{367}{55} \text{ or } 6\frac{37}{55} \end{aligned}$$

The mixed number form of $6.\overline{672}$ is $6\frac{37}{55}$.

ANSWER

$$6\frac{37}{55}$$

Example 2:

Convert $0.\overline{726}$ to a fraction. Write the answer in simplest form.

STEP 1

Set the repeating decimal equal to a variable. Use x to represent the decimal.

$$x = 0.\overline{726}$$

STEP 2

Multiply the equation by 1000.

$$\begin{aligned}x &= 0.\overline{726} \\ 1000x &= 1000(0.\overline{726}) \\ 1000x &= 726.\overline{726}\end{aligned}$$

STEP 3

Subtract the equations.

$$\begin{array}{r} 1000x = 726.\overline{726} \\ - \quad x = 0.\overline{726} \\ \hline 999x = 726 \end{array}$$

STEP 4

Solve the resulting equation. Isolate x by dividing each side of the equation by 999. Simplify the result.

$$\begin{aligned}999x &= 726 \\ \frac{999x}{999} &= \frac{726}{999} \\ x &= \frac{242}{333}\end{aligned}$$

ANSWER

$$\frac{242}{333}$$

Guided Practice:

Convert each decimal to a fraction:

$$.\overline{24} =$$

$$\overline{.567} =$$

Independent Practice: Convert each decimal to a fraction. *Show your work!*

1. $\overline{.916} =$

2. $4.\overline{375} =$

3. $3.\overline{726} =$

4. $1.\overline{26} =$

Additional Help:

<http://www.mathsisfun.com/convertng-decimals-fractions.html>

<http://www.basic-mathematics.com/convertng-repeating-decimals-to-fractions.html>

<https://www.youtube.com/watch?v=jVzjopXPMIc>