Exponent Rules

Power or **Exponent**- number of times a given number is used as a factor in repeated multiplication

$a^3 = a \times a \times a$ $(-2)^4 = (-2)(-2)(-2)(-2) = 16$

Negative Exponent - Reciprocal of that number with a **positive** exponent

$a^{-n} = \frac{1}{a^n}$ $(5)^{-3} = \frac{1}{(-5)^3} = \frac{1}{(-5)\cdot(-5)\cdot(-5)} = -\frac{1}{125}$

Zero Power - Any number raised to the power of zero is 1

$a^0 = 1$ $7^0 = 1$

Product of Powers Property: To multiply two monomial expressions that have the same base, add the exponents

$a^b \cdot a^c = a^{b+c}$ $9^6 \cdot 9^8 = 9^{6+8} = 9^{14}$

Power of a Product Property: To multiply two monomial expressions that have the same exponent, multiply the bases

$a^c \cdot b^c = ab^c$ $(8)^3 \cdot (3)^3 = (8 \cdot 3)^3 = (24)^3$

Power of Product Property: To find the power of a power, multiply the exponents

$(a^b)^c = a^{b \cdot c}$ $(5^3)^4 = 5^{3 \cdot 4} = 5^{12}$

Quotient of Powers Property: To divide the powers, subtract the exponents

$\frac{a^b}{a^c} = a^{b-c}$ $\frac{(-7)^{-3}}{(-7)^{-6}} = (-7)^{-3-(-6)} = (-7)^3$

Now put it all together:

PROBLEM

Simplify (use only positive exponents):

$$-x^5y^{-3}\left(-4x^{-3}y^{-1}\right)^{-4}$$

STEP 1

Apply the power of a product property.

$$-x^5y^{-3}\left(-4x^{-3}y^{-1}\right)^{-4} = -x^5y^{-3}(-4)^{-4}\left(x^{-3}\right)^{-4}\left(y^{-1}\right)^{-4}$$

STEP 2

Apply the power of a power property.

$$-x^5y^{-3}(-4)^{-4}\left(x^{-3}\right)^{-4}\left(y^{-1}\right)^{-4} = -x^5y^{-3}(-4)^{-4}\left(x^{-3\cdot-4}\right)\left(y^{-1\cdot-4}\right)$$
$$= -x^5y^{-3}(-4)^{-4}\left(x^{12}\right)\left(y^4\right)$$

STEP 3

Apply the associative and commutative property to group the constants and the like variables together.

$$-x^5y^{-3}(-4)^{-4}\left(x^{12}\right)\left(y^4\right) = (-1)\left(x^5y^{-3}\right)(-4)^{-4}\left(x^{12}\right)\left(y^4\right)$$
$$= (-1)(-4)^{-4}\left(x^5 \cdot x^{12}\right)\left(y^{-3} \cdot y^4\right)$$

STEP 4

Apply the product of powers property.

$$-1(-4)^{-4}\left(x^5 \cdot x^{12}\right)\left(y^{-3} \cdot y^4\right) = -1 \cdot (-4)^{-4}\, x^{5+12}y^{(-3)+4}$$
$$= -1 \cdot (-4)^{-4}\, x^{17}y$$

STEP 5

Use the definition of a negative exponent to simplify the expression.

$$-1 \cdot (-4)^{-4}\, x^{17}y = \frac{-1 \cdot x^{17}y}{(-4)^4}$$
$$= -\frac{x^{17}}{256}$$

PROBLEM

Simplify: $\frac{\left(-5m^{-6}\right)^{-3}}{\left(5m^2\right)m^{-2}}$

STEP 1

Simplify the numerator by applying properties of exponents.

$$\left(-5m^{-6}\right)^{-3} = (-5)^{-3}\left(m^{-6}\right)^{-3}$$
$$= (-5)^{-3}\, m^{(-6)(-3)}$$
$$= (-5)^{-3}\, m^{18}$$

STEP 2

Simplify the denominator by applying the product of a power property.

$$\left(5m^2\right)m^{-2} = 5m^{2+(-2)}$$
$$= 5m^0$$
$$= 5 \cdot 1$$
$$= 5$$

STEP 3

Rewrite the expression with the simplified numerator and denominator.

$$\frac{\left(-5m^{-6}\right)^{-3}}{\left(5m^2\right)m^{-2}} = \frac{(-5)^{-3}\, m^{18}}{5}$$

STEP 4

Simplify the expression.

$$\frac{(-5)^{-3}\, m^{18}}{5} = \frac{m^{18}}{(-5)^3 \cdot 5}$$
$$= \frac{m^{18}}{-125 \cdot 5}$$
$$= -\frac{m^{18}}{625}$$